Chapter 10. Straight Lines

Question-1

Determine the equation of the straight line passing through the point (-1, -2) and having slope 4/7.

Solution:

The point – slope form is $y - y_1 = m(x - x_1)$

$$y + 2 = (4/7)(x + 1)$$
$$7y + 14 = 4x + 4$$
$$4x - 7y = 10$$

Question-2

Determine the equation of the line with slope 3 and y – intercept 4.

Solution:

The slope – intercept form is $y = mx + c$.

Therefore the equation of the straight line is $y = 3m + 4$.

Question-3

A straight line makes an angle of 45° with x – axis and passes through the point (3, -3). Find its equation.

Solution:

$m = \tan 45^{\circ} = 1$

The slope – intercept form is $y = mx + c$.

$(-3) = 1(3) + c$

$c = -6$

Therefore the equation of the straight line is $y = x - 6$.

Question-3

A straight line makes an angle of 45° with x – axis and passes through the point (3, -3). Find its equation.

Solution:

$m = \tan 45^{\circ} = 1$

The slope – intercept form is $y = mx + c$.

$(-3) = 1(3) + c$

$c = -6$

Therefore the equation of the straight line is $y = x - 6$.

Question-4

Find the equation of the straight line joining the points (3, 6) and (2, -5).

Solution:

The equation of a straight line passing through two points is $\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$.

Substituting the points (3, 6) and (2, -5),

$\frac{y-6}{6+5} = \frac{x-3}{3-2}$

$\frac{y-6}{11} = \frac{x-3}{1}$

$y - 6 = 11(x - 3)$

$y - 6 = 11x - 33$

$11x - y = 27$ is the required equation of the straight line.

The equation of a straight line passing through two points is $\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$.

Substituting the points (3, 6) and (2, -5),

$\frac{y-6}{6+5} = \frac{x-3}{3-2}$

$\frac{y-6}{11} = \frac{x-3}{1}$

$y - 6 = 11(x - 3)$

$y - 6 = 11x - 33$

$11x - y = 27$ is the required equation of the straight line.

Question-5

Find the equation of the straight line passing through the point (2, 2) and having intercepts whose sum is 9.

Solution:

The intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

$a + b = 9$ (Given) ....(i)

$\frac{x}{a} + \frac{y}{b} = 1$

Since (2, 2) lies on the equation, $\frac{2}{a} + \frac{2}{b} = 1$

$2(a + b) = ab$

$ab = 18$ (from i)

$a = 18/b$

Substituting in (i)

$18 + b^2 = 9b$

$b^2 - 9b + 18 = 0$

$b^2 - 6b - 3b + 18 = 0$

$b(b - 6) - 3(b - 6) = 0$

$(b - 3)(b - 6) = 0$

$b = 3$ or 6

Therefore $a = 6$ or 3.

Therefore the required equation of straight line is $\frac{x}{3} + \frac{y}{6} = 1$ or $\frac{x}{6} + \frac{y}{3} = 1$.

Question-6

Find the equation of the straight line whose intercept on the x-axis is 3 times its intercept on the y-axis and which passes through the point (-1, 3).

Solution:

The intercept form is $\frac{x}{a}+\frac{y}{b}=1$ where 'a' and 'b' are x and y intercepts respectively.

$a = 3b$ (Given) ……………….(i)

$\frac{x}{3b}+\frac{y}{b} = 1$

Since (2, 2) lies on the equation, $\frac{2}{3b}+\frac{2}{b} = 1$

$b = \frac{2}{3}+\frac{2}{1}=\frac{2+6}{3}=\frac{8}{3}$

$\therefore$ a = 8.

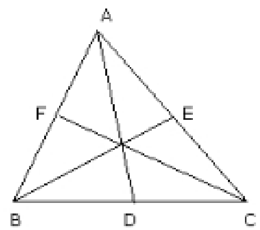
$\therefore$ The required equation of straight line is $\frac{x}{8}+\frac{3y}{8} = 1$ i.e., x + 3y = 8.

Question-7

Find the equations of the medians of the triangle formed by the point (2, 4), (4, 6) and (-6, -10).

Solution:

Let A(2, 4), B(4, 6) and C(-6, -10) be the given vertices of a Δ ABC. D, E, F re the mid-points of the sides BC, CA, AB respectively.



$\therefore D = \left(\frac{4-6}{2},\frac{6-10}{2}\right) = \left(\frac{-2}{2},\frac{-4}{2}\right) = (-1, -2)$

$E = \left(\frac{2-6}{2},\frac{4-10}{2}\right) = \left(\frac{-4}{2},\frac{-6}{2}\right) = (-2, -3)$

$F = \left(\frac{2+4}{2},\frac{4+6}{2}\right) = \left(\frac{6}{2},\frac{10}{2}\right) = (3, 5)$

∴ Equation of AD is

$\frac{y-2}{2+1} = \frac{x-4}{4+2}$

6(y - 2) = 3(x - 4)

2(y - 2) = (x - 4)

2y - 6 = x – 4

2y – x = 0

∴ Equation of BE is

$\frac{y-6}{6+3} = \frac{x-4}{4+2}$

6(y - 6) = 9(x - 4)

2(y - 6) = 3(x - 4)

2y - 12 = 3x - 12

3y - 3x = 0

∴ Equation of CF is

$\frac{y+10}{-10-5} = \frac{x+6}{-6-3}$

-9(y + 10) = -15(x + 6)

3(y + 10) = 5(x + 6)

3y + 30 = 5x + 30

3y – 5x = 0

Question-8

Find the length of the perpendicular from (3, 2) to the straight line 3x + 2y + 1 = 0.

Solution:

The perpendicular distance from (x_1, y_1) to the straight line ax + by + c = 0 is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.∴ The length of the perpendicular from (3, 2) to the straight line 3x + 2y + 1 = 0 is $\left|\frac{3(3) + 2(2) + 1}{\sqrt{3^2 + 2^2}}\right| = \frac{14}{\sqrt{13}}$.--

Question-9

The portion of straight line between the axes is bisected at the point (-3, 2). Find its equation.

Solution:

The intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

The straight line make the x intercept OA = a, and y intercept OB = b.

Then, A is (0, a) and B is (b, 0).

$\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

Mid-point of AB is $\left(\frac{b}{2}, \frac{a}{2}\right)$ = (-3, 2)

b = -6 and a = 4.

∴ The equation of straight line is $\frac{x}{4} + \frac{y}{-6} = 1$.

Question-10

Find the equation of the diagonals of quadrilateral whose vertices are (1, 2), (-2, -1), (3, 6) and (6, 8).

Solution:

Let A(1, 2), B(-2, -1), C(3, 6) and D(6, 8) be the vertices of quadrilateral ABCD.

Equation of the diagonal AC is $\frac{y-2}{2-6} = \frac{x-1}{1-3}$

$-2(y - 2) = -4(x - 1)$

$-2y + 4 = -4x + 1$

$4x - 2y + 3 = 0$

Equation of the diagonal BD is $\frac{y+1}{-1-8} = \frac{x+2}{-2-8}$

$-16(y + 1) = -9(x + 2)$

$-16y - 16 = -9x - 18$

$9x - 16y + 2 = 0$

Question-11

Find the equation of the straight line, which cut of intercepts on the axes whose sum and product are 1 and −6 respectively.

Solution:

The intercept form is $\frac{x}{a}+\frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

$a + b = 1$ ………….(i)

$ab = -6$ ………….(ii)

$(a - b)^2 = 4ab - (a + b)^2 = 4(-6) - (1)^2 = -24 - 1 = 25$

$a - b = 5$ ………….(iii)

Adding (i) and (iii)

$2a = 6$

$a = 3$

$\therefore b = -2$

$\therefore$ The equation of the straight line is $\frac{x}{3}+\frac{y}{-2} = 1$.

Question-12

Find the intercepts made by the line 7x + 3y − 6 = 0 on the coordinate axis.

Solution:

If y = 0 then x = 6/7

If x = 0 then y = 6/3 = 2

x - intercept is 6/7 and y − intercept is 2.

Question-13

What are the points on x-axis whose perpendicular distance from the straight line $\frac{x}{3} + \frac{y}{4} = 1$ is 4?

Solution:

$\frac{x}{3} + \frac{y}{4} = 1$

$4x + 3y = 12$

Any point on x – axis will have y coordinate as 0.

Let the point on x-axis be $P(x_1, 0)$.

The perpendicular distance from the point P to the given straight line is

$\left|\frac{4(x_1) + 3(0) - 12}{\sqrt{4^2 + 3^2}}\right| = 4$

$\left|\frac{4x_1 - 12}{5}\right| = 4$ or $\left|\frac{4x_1 - 12}{5}\right| = -4$

$4x_1 - 12 = 20$ or $4x_1 - 12 = -20$

$4x_1 = 32$ or $4x_1 = -8$

$x_1 = 8$ or $x_1 = -2$

Thus the required points are (8, 0) and (-2, 0).

Question-14

Find the distance of the line 4x – y = 0 from the point (4, 1) measured along the straight line making an angle of 135° with the positive direction of the x-axis.

Solution:

$m = \tan 135^\circ = -1$

Equation of a straight line having slope m = -1 and passing through (4, 1) is

$y - y_1 = m(x - x_1)$

$y - 1 = (-1)(x - 4)$

$y - 1 = -x + 4$

$x + y = 5$ ……………….(i)

$4x - y = 0$ ……………….(ii)

Solving (i) and (ii),

x = 1 and y = 4

∴ The distance between the points (1, 4) and (4, 1) is

$= \sqrt{(4-1)^2 + (1-4)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = 3\sqrt{2}$ units

Question-15

Find the angle between the straight lines 2x + y = 4 and x + 3y = 5

Solution:

Slope of the line 2x + y = 4 is m_1 = -2.

and slope of the line x + 3v = 5 is m_2 = -1/3

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1m_2}\right| = \tan^{-1}\left|\frac{-2+\frac{1}{3}}{1+(-2)(-\frac{1}{3})}\right| = \tan^{-1}\left|\frac{\frac{-6+1}{3}}{1+\frac{2}{3}}\right| = \tan^{-1}\left|\frac{\frac{-5}{3}}{\frac{5}{3}}\right| = \tan^{-1}(-1) = 135^{o}$$

Question-16

Show that the straight lines 2x + y = 5 and x – 2y = 4 are at right angles.

Solution:

Slope of the line 2x + y = 5 is m_1 = -2.

and slope of the line x - 2y = 4 is m_2 = ½

m_1 m_2 = -2(1/2) = -1

∴ The two straight lines are at right angles.

Question-17

Find the equation of the straight line passing through the point (1, -2) and parallel to the straight line 3x + 2y – 7 = 0.

Solution:

The straight line parallel to 3x + 2y – 7 = 0 is of the form 3x + 2y + k = 0 ....(i)

The point (1, -2) satisfies the equation (i)

Hence 3(1) + 2(-2) + k = 0

⇒ 3 - 4 + k = 0⇒ k = 1

∴ 3x + 2y + 1 = 0 is the equation of the required straight line.

Question-18

Find the equation of the straight line passing through the point (2, 1) and perpendicular to the straight line x + y = 9.

Solution:

The equation of the straight line perpendicular to the straight line x + y = 9 is of the form x - y + k = 0.

The point (2, 1) lies on the straight line x - y + k = 0.

∴ 2 - 1 + k = 0

k = -1

∴ The equation of the required straight line is x - y – 1 = 0.

Question-19

Find the point of intersection of the straight lines 5x + 4y – 13 = 0 and 3x + y – 5 = 0.

Solution:

Let (x_1, y_1) be the point of intersection. Then (x_1, y_1) lies on both the straight lines.

$\therefore 5x_1 + 4y_1 - 13 = 0$ ……………..(i)

$3x_1 + y_1 - 5 = 0$ ……………..(ii)

(ii) × 4 $12x_1 + 4y_1 - 20 = 0$ ……………(iii)

(i) – (iii) $-7x_1 + 7 = 0$

$x_1 = 1$

Substituting $x_1 = 1$ in (i) $5(1) + 4y_1 - 13 = 0$

$5 + 4y_1 - 13 = 0$

$4y_1 - 8 = 0$

$y_1 = 2$

$\therefore$ The point of intersection is (1, 2).

Question-20

If the two straight lines 2x – 3y + 9 = 0, 6x + ky + 4 = 0 are parallel, find k.

Solution:

The two given equations are parallel.

$2x - 3y + 9 = 0$ ……………..(i)

$6x + ky + 4 = 0$ ………………(ii)

$\therefore$ The coefficients of x and y are proportional $\frac{2}{6} = \frac{-3}{k}$ $\therefore$ k = -9.

Question-21

Find the distance between the parallel lines 2x + y – 9 = 0 and 4x + 2y + 7 = 0.

Solution:

The distance between the parallel lines is $\left|\frac{-9-\frac{7}{2}}{\sqrt{2^2+1^2}}\right| = \left|\frac{\frac{-25}{2}}{\sqrt{5}}\right| = \left|\frac{-25}{2\sqrt{5}}\right| = \frac{5\sqrt{5}}{2}$ units

Question-22

Find the values of p for which the straight lines 8px + (2 − 3p)y + 1 = 0 and px + 8y − 7 = 0 are perpendicular to each other.

Solution:

Slope of the line 8px + (2 − 3p)y + 1 = 0 is $m_1 = 8p/(2 - 3p)$.

and slope of the line px + 8y − 7 = 0 is $m_2 = -p/8$

$m_1 m_2 = -1$

$\frac{8p}{2-3p} \times \frac{-p}{8} = -1$

$-p^2 = -(2 - 3p)$

$p^2 + 3p - 2 = 0$

$p^2 + 2p + p - 2 = 0$

$(p + 1)(p - 2) = 0$

∴ The two straight lines are at right angles.

Question-23

Find the equation of the straight line which passes through the intersection of the straight lines 2x + y = 8 and 3x − 2y + 7 = 0 and is parallel to the straight line 4x + y − 11 = 0.

Solution:

$2x + y - 8 = 0$ ....(i)

$3x - 2y + 7 = 0$ ....(ii)

(i) × 2

$4x + 2y - 16 = 0$ ....(iii)

(ii) + (iii)

$7x - 9 = 0$

$x = 9/7$

Substituting x = 9/7 in (i)

$2(9/7) + y - 8 = 0$

$18/7 + y - 8 = 0$

$y = 8 - 18/7 = 38/7$

Point of intersection of 2x + y = 8 and 3x − 2y + 7 = 0 is (9/7, 38/7).

The equation of lines parallel the straight line 4x + y − 11 = 0 is 4x + y + k = 0.

$4\left(\frac{9}{7}\right) + \frac{38}{7} + k = 0$

$k = -\frac{74}{7}$ ∴ The required equation of straight line is $4x + y - \frac{74}{7} = 0$ i.e., $28x + 7y - 74 = 0$.

Question-24

Find the equation of the straight line passing through intersection of the straight lines 5x − 6y = 1 and 3x + 2y + 5 = 0 and perpendicular to the straight line 3x − 5y + 11 = 0.

Solution:

Equation of line through the intersection of straight lines 5x − 6y = 1 and 3x + 2y + 5 = 0 is

$(5x - 6y - 1) + k(3x + 2y + 5) = 0$

$(5 + 3k)x + (-6 + 2k)y + (-1 + 5k) = 0$

Slope of the above equation is $-(5 + 3k)/(-6 + 2k)$.

Above equation is perpendicular to 3x − 5y + 11 = 0.

Question-25

Find the equation of the straight line joining (4, -3) and the intersection of the straight lines 2x − y + 7 = 0 and x + y − 1 = 0.

Solution:

$2x - y + 7 = 0$ ……………….(i)

$x + y - 1 = 0$ ……………….(ii)

(i) + (ii)

$3x + 6 = 0$

$x = -2$

Substituting x = -2 in (i)

$2(-2) - y + 7 = 0$

$-4 - y + 7 = 0$

$y = 3$

The equation of the line joining (4, -3) and (-2, 3) is

$\frac{y+3}{-3-3} = \frac{x-4}{4+2}$

$6(y + 3) = -6(x - 4)$

$y + 3 + x - 4 = 0$

$x + y = 1$

Question-26

Find the equation of the straight line joining the point of the intersection of the straight lines $3x + 2y + 1 = 0$ and $x + y = 3$ to the point of intersection of the straight lines $y - x = 1$ and $2x + y + 2 = 0$.

Solution:

$3x + 2y + 1 = 0$ ....(i)

$x + y = 3$ ....(ii)

(ii) × -2

$-2x - 2y + 6 = 0$ ....(iii)

(i) + (iii)

$x + 7 = 0$

$x = -7$

Substituting $x = -7$ in (ii)

$x + y - 3 = 0$

$-7 + y - 3 = 0$

$y = 10$

∴ The point of intersection of lines $3x + 2y + 1 = 0$ and $x + y = 3$ is $(-7, 10)$.

$y - x - 1 = 0$ ....(i)

$2x + y + 2 = 0$ ....(ii)

(i) − (ii)

$-3x - 3 = 0$

$x = -1$

Substituting $x = -1$ in (ii)

$y + 1 - 1 = 0$

$y = 0$

∴ The point of intersection of lines $y - x = 1$ and $2x + y + 2 = 0$ is $(-1, 0)$.

The equation of the line joining $(-7, 10)$ and $(-1, 0)$ is

$\frac{y-10}{10-0} = \frac{x+7}{-7+1}$

$-6(y - 10) = 10(x + 7)$

$-3(y - 10) = 5(x + 7)$

$-3y + 30 = 5x + 35$

$5x + 3y + 5 = 0.$

∴ The required equation of straight line is $5x + 3y + 5 = 0$.

Question-27

Show that the angle between $3x + 2y = 0$ and $4x - y = 0$ is equal to the angle between $2x + y = 0$ and $9x + 32y = 41$.

Solution:

Slope of the line $3x + 2y = 0$ is $m_1 = -3/2$.

Slope of the line $4x - y = 0$ is $m_2 = 4$.

Angle between the straight line $3x + 2y = 0$ and $4x - y = 0$ is $\tan\theta_1$

$$= \left|\frac{\frac{-3}{2}-4}{1+\left(\frac{-3}{2}\right)\times 4}\right| = \left|\frac{\frac{-3-8}{2}}{1-6}\right| = \left|\frac{\frac{-11}{2}}{-5}\right| = 11/10$$

Slope of the line $2x + y = 0$ is $m_1 = -2$.

Slope of the line $9x + 32y = 41$ is $m_2 = -9/32$.

Angle between the straight line $2x + y = 0$ and $9x + 32y = 41$ is $\tan\theta_2 =$

$$\left|\frac{-2+\frac{9}{32}}{1+(-2)\left(-\frac{9}{32}\right)}\right|$$

$$= \left|\frac{\frac{-64+9}{32}}{1+\frac{18}{32}}\right| = \left|\frac{\frac{-55}{32}}{\frac{50}{32}}\right| = 11/10 \therefore \tan\theta_1 = \tan\theta_2 \therefore \theta_1 = \theta_2$$

Question-28

Show that the triangle whose sides are $y = 2x + 7$, $x - 3y - 6 = 0$ and $x + 2y = 8$ is right angled. Find its other angles.

Solution:

Slope of the line $y = 2x + 7$ is $m_1 = 2$.

Slope of the line $x - 3y - 6 = 0$ is $m_2 = 1/3$.

Slope of the line $x + 2y = 8$ is $m_3 = -1/2$.

Angle between the straight line $y = 2x + 7$ and $x - 3y - 6 = 0$ is

$$\tan\theta_1 = \left|\frac{2-\frac{1}{3}}{1+(2)\left(\frac{1}{3}\right)}\right| = \left|\frac{\frac{6-1}{3}}{\frac{3+2}{3}}\right| = \left|\frac{\frac{5}{3}}{\frac{5}{3}}\right| = 1 \ \backslash\ \theta_1 = 45^\circ$$

Angle between the straight line $x - 3y - 6 = 0$ and $x + 2y = 8$ is

$$\tan\theta_2 = \left|\frac{\frac{1}{3}+\frac{1}{2}}{1+\left(\frac{1}{3}\right)\left(-\frac{1}{2}\right)}\right| = \left|\frac{\frac{2+3}{6}}{\frac{6-1}{6}}\right| = \left|\frac{\frac{5}{6}}{\frac{5}{6}}\right| = 1$$

$\therefore \theta_2 = 45^\circ$

Angle between the straight line $x + 2y = 8$ and $y = 2x + 7$ is

$$\tan\theta_3 = \left|\frac{2+\frac{1}{2}}{1+(2)\left(-\frac{1}{2}\right)}\right| = \left|\frac{\frac{4+1}{2}}{\frac{2-2}{2}}\right| = \left|\frac{\frac{5}{2}}{\frac{0}{2}}\right| \qquad \theta_3 = 90^\circ$$

$\therefore$ The triangle is right angled isosceles.

Question-29

Show that the straight lines 3x + y + 4 = 0, 3x + 4y - 15 = 0 and 24x − 7y − 3 = 0 form an isosceles triangle.

Solution:

Slope of the line 3x + y + 4 = 0 is m_1 = -3.
Slope of the line 3x + 4y - 15 = 0 is m_2 = -3/4.
Slope of the line 24x − 7y − 3 = 0 is m_3 = 24/7.
Angle between the straight line 3x + y + 4 = 0 and 3x + 4y - 15 = 0 is

$$\tan\theta_1 = \left|\frac{-3+\frac{3}{4}}{1+(-3)\left(-\frac{3}{4}\right)}\right| = \left|\frac{\frac{-12+3}{4}}{\frac{4+9}{4}}\right| = \left|\frac{9}{13}\right| = 9/13$$

Angle between the straight line 3x + 4y - 15 = 0 and 24x − 7y − 3 = 0 is

$$\tan\theta_2 = \left|\frac{-\frac{3}{4}-\frac{24}{7}}{1+\left(-\frac{3}{4}\right)\left(\frac{24}{7}\right)}\right| = \left|\frac{\frac{-21-96}{28}}{\frac{28-72}{28}}\right| = \left|\frac{\frac{-117}{28}}{\frac{-44}{28}}\right| = 117/44$$

Angle between the straight line 24x − 7y − 3 = 0 and 3x + y + 4 = 0 is

$$\tan\theta_3 = \left|\frac{\frac{24}{7}+3}{1+\left(\frac{24}{7}\right)(-3)}\right| = \left|\frac{\frac{24+21}{7}}{\frac{7-72}{7}}\right| = \left|\frac{\frac{45}{7}}{\frac{-65}{7}}\right| = 9/13$$

$\tan\theta_1 = \tan\theta_3$
∴ The triangle is isosceles.

Question-30

Show that the straight lines 3x + 4y = 13, 2x − 7y + 1 = 0 and 5x − y = 14 are concurrent.

Solution:

3x + 4y = 13 ....(i)
2x − 7y + 1 = 0 ....(ii)
5x − y = 14 ....(iii)
(iii) × 4
20x − 4y = 56 ....(iv)
(i) + (iv)
23x = 69
x = 3
Substitute x = 3 in (iii)
15 − y = 14
y = 1
∴ The point of intersection is (3, 1).
3(3) + 4(1) = 13
9 + 4 = 13
The point (3, 1) satisfies equation (i). Hence they are concurrent.

Question-31

Find 'a' so that the straight lines $x - 6y + a = 0$, $2x + 3y + 4 = 0$ and $x + 4y + 1 = 0$ may be concurrent.

Solution:

$x - 6y + a = 0$ ....(i)

$2x + 3y + 4 = 0$ ....(ii)

$x + 4y + 1 = 0$ ....(iii)

2 × (iii)

$2x + 8y + 2 = 0$ ....(iv)

(ii) − (iv)

$-5y + 2 = 0$

$y = 2/5$

Substituting $y = 2/5$ in (iii)

$x + 4(2/5) + 1 = 0$

$x = -13/5$

Substituting (-13/5, 2/5) in (i),

$\frac{-13}{5} - 6\times \frac{2}{5} + a = 0$

$-25 + 5a = 0$

$a = 5$

Question-32

Find the values of 'a' for which the straight lines $x + y - 4 = 0$, $3x + 2 = 0$ and $x - y + 3a = 0$ are concurrent.

Solution:

$x + y - 4 = 0$ ....(i)

$3x + 2 = 0$ ....(ii)

$x - y + 3a = 0$ ....(iii)

$x = -2/3$ ....(iv)

$(-2/3) + y - 4 = 0$

$y = 4 + 2/3 = 14/3$

Substituting (-2/3, 14/3) in (iii),

$(-2/3) - (14/3) + 3a = 0$

$(-16/3) + 3a = 0$

$a = 16/9$

Question-33

Find the coordinates of the orthocentre of the triangle whose vertices are the points (-2, -1), (6, -1) and (2, 5).

Solution:

Let A(-2, -1), B(6, -1) and C(2, 5) be the vertices of the triangle ABC.

Line AB is $\frac{y+1}{-1+1} = \frac{x+2}{-2-6}$

$x + 2 = 0$ ....(i)

Line perpendicular to $x + 2 = 0$ is $x + k = 0$

If it passes through (2, 5), then $2 + k = 0$, $k = -2$

$\therefore x - 2 = 0$ is one altitude. ....(ii)

Line BC is $\frac{y+1}{-1-5} = \frac{x-6}{6-2}$

$\frac{y+1}{-6} = \frac{x-6}{4}$

$2(y + 1) = -3(x - 6)$

$2y + 3x = 16$ ....(iii)

Line perpendicular to $2y + 3x = 16$ is $2x - 3y + k = 0$.

If it passes through (-2, -1), then $2(-2) - 3(-1) + k = 0$ i.e., $-4 + 3 + k = 0$ i.e., $k = 1$

$\therefore 2x - 3y + 1 = 0$ is another altitude. ....(iv)

Solving (ii) and (iv)

$x = 2$

$2(2) - 3y + 1 = 0$

$4 - 3y + 1 = 0$

$-3y = -5$

$y = 5/3$

$\therefore$ Orthocentre is (2, 5/3).

Question-34

If $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, show that $a^3 + b^3 + c^3 = 3abc$.

Solution:

The condition for three lines concurrency is $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$a(cb - a^2) - b(b^2 - ca) + c(ab - c^2) = 0$

$abc - a^3 - b^3 + abc + abc - c^3 = 0$

$abc - a^3 - b^3 + abc + abc - c^3 = 0$

\ $a^3 + b^3 + c^3 = 3abc$.

Question-35

Find the coordinates of the orthocentre of the triangle formed by the straight lines x + y − 1 = 0, x + 2y − 4 = 0 and x + 3y − 9 = 0.

Solution:

Let the equation of sides AB, BC and CA of a Δ BC be represented by

$x + y - 1 = 0$ ....(i)

$x + 2y - 4 = 0$ ....(ii)

$x + 3y - 9 = 0$ ....(iii)

(i) × 2

$2x + 2y - 2 = 0$ ....(iv)

(ii) - (iv)

$-x - 2 = 0$

$x = -2$

Substituting x = -2 in (ii)

$-2 + 2y - 4 = 0$

$2y = 6$

$y = 3$

The vertex A is (-2, 3).

The equation of the straight line CA is x + 3y − 9 = 0. The straight line perpendicular to its is of the form 3x - y + k = 0 ....(v)

A(-2, 3) satisfies the equation (v)

$\therefore 3x - y + k = 0$

$3(-2) - 3 + k = 0$

$k = 9$

The equation of AD is 3x - y + 9 = 0. ....(vi)

(ii) − (iii)

$-y + 5 = 0$

$y = 5$

Substituting y = 5 in (ii)

$x + 2(5) - 4 = 0$

$x + 10 - 4 = 0$

$x = -6$

The vertex C is (-6, 5).

The equation of the straight line AB is x + y − 1 = 0. The straight line perpendicular to its is of the form x - y + k = 0 ....(vii)

C(-6, 5) satisfies the equation (vii)

$\therefore x - y + k = 0$

$-6 - 5 + k = 0$

$k = 11$

The equation of CE is $x - y + 11 = 0$. ....(viii)

(vi) – (viii)

$2x - 2 = 0$

$x = 1$

Substituting $x = 1$ in (viii)

$1 - y + 11 = 0$

$y = 12$

$\therefore$ The orthocentre O is (1, 12).

Question-36

The equation of the sides of a triangle are x + 2y = 0, 4x + 3y = 5 and 3x + y = 0. Find the coordinates of the orthocentre of the triangle.

Solution:

Let the equation of sides AB, BC and CA of a Δ BC be represented by

$x + 2y = 0$ ....(i)

$4x + 3y = 5$ ....(ii)

$3x + y = 0$ ....(iii)

Substituting $x = -2y$ (ii)

$4(-2y) + 3y = 5$

$-8y + 3y = 5$

$y = -1$

$\therefore x = 2$

The vertex B is (2, -1).

The equation of the straight line AC is $3x + y = 0$. The straight line perpendicular to its is of the form $x - 3y + k = 0$ ....(iv)

B(2, -1) satisfies the equation (iv)

$\therefore 2 - 3(-1) + k = 0$

$2 + 3 + k = 0$

$k = -5$

The equation of BD is $x - 3y - 5 = 0$. ……………..(v)

Substituting $x = -2y$ (iii)

$3(-2y) + y = 0$

$-6y + y = 0$

$-5y = 0$

$y = 0$

$\therefore x = 0$

The vertex A is (0, 0).The equation of the straight line BC is $4x + 3y = 5$. The straight line perpendicular to its is of the form $3x - 4y + k = 0$ ……………….. (vi)

A(0, 0) satisfies the equation (vi)

$\therefore 3(0) - 4(0) + k = 0$

$k = 0$

The equation of AE is $3x - 4y = 0$ ……………..(vii)

$3 \times$ (v) $-$ (vii)

$3x - 9y - 15 = 0$

$\therefore -5y - 15 = 0$

$y = -15/5 = -3$

Substitute $y = -3$ in (vii)

$3x - 4(-3) = 0$

$3x + 12 = 0$

$x = -4 \therefore$ The orthocentre O is (-4, -3).